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COMPUTING SHORTEST DISTANCE OF HAMILTONIAN FUZZY CYCLES USING EIGEN VALUES

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Abstract: - Decomposing the complete fuzzy graph into Hamiltonian fuzzy cycles is called the Hamiltonian fuzzy decomposition. The complete fuzzy graph of (2n+1) vertices can be decomposed into n Hamiltonian fuzzy cycles and the complete fuzzy graph of 2n vertices can be decomposed into (n-1) Hamiltonian fuzzy cycles. In this paper we discuss about, which one of these Hamiltonian fuzzy cycles having the shortest distance by using the Eigen values.

Keywords: - Complete fuzzy graph, Covariance matrix, Eigen value, Fuzzy graph, Hamiltonian fuzzy cycle.

I. INTRODUCTION

Graph theory is a delightful playground for the exploration of proof techniques in discrete Mathematics and its results have applications in many areas of the computing, social and natural sciences. A graph is a convenient way of representing information involving relationship between objects. The objects are represented by vertices and the relations by edges. When there is vagueness in the description of the objects or in its relationships or in both, it is natural that we need to design a fuzzy graph model. Fuzzy graph is also a symmetric binary fuzzy relation on a fuzzy subset. The concept of fuzzy sets and fuzzy relations was introduced by L.A. Zadeh in 1965. In 1975 Rosenfeld introduced the notion of fuzzy graph and several analogs of graph theoretic concepts such as path, cycles and connectedness. The concept of decomposition of regular graphs was introduced by Klas Markstrom. Decomposition of complete graphs into Hamiltonian cycles and Decomposition of fuzzy graphs introduced by Dr. G. Nirmala and M. Vijaya. Here we discuss how to compute the shortest distance of Hamiltonian fuzzy cycles using Eigen values.

II. ALGORITHM

1. 1. Write the membership value of each vertex in the first row of a matrix for n cycles.

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2. Write the membership value of each edge in the second row of the matrix for n cycles. As a result we get a 2xn matrix.
3. Convert this matrix as a covariance matrix of order 2x2.

$$C = \begin{pmatrix} cov(x, x) & cov(x, y) \\ cov(y, x) & cov(y, y) \end{pmatrix}$$

4. Find the Eigen value of C
5. Consider only the added value of the Eigen value
6. Comparing the added values of the Eigen value we have to select the minimum value.
7. This minimum value is the shortest distance among the Hamiltonian fuzzy cycle.

III. ADDED VALUE OF EIGEN VALUE

Since the covariance matrix is a 2x2 matrix, there are 2 Eigen values out of which one is obtained in general, from the expression $-B+\sqrt{(B^2 - 4AC)}/2A$ and the other is obtained from $-B-\sqrt{(B^2 - 4AC)}/2A$. The value obtained from the first expression is called added Eigen value.

3.1 Shortest distances measured in Hamiltonian fuzzy cycle using Eigen values of (2n+1) vertices

The shortest cycle from Hamiltonian fuzzy cycles are illustrated by the following examples. If the fuzzy graph has 3 vertices then it has only one cycle. Suppose the given fuzzy graph is complete fuzzy graph with 5 vertices we decompose that into 2 Hamiltonian fuzzy cycles. Let the membership values of vertices of K_5 are (0.3 0.4 0.8 0.9 0.7). Then the corresponding membership value of Hamiltonian fuzzy cycles are (0.3 0.4 0.8 0.9 0.7) and (0.3 0.8 0.7 0.4 0.9). The first fuzzy cycle form the following matrix

$$\begin{matrix} V \\ E \end{matrix} \begin{pmatrix} 0.3 & 0.4 & 0.8 & 0.9 & 0.7 \\ 0.3 & 0.4 & 0.8 & 0.7 & 0.3 \end{pmatrix}$$

The first row values are membership values of vertices and the second row values are membership values of edges of the first Hamiltonian fuzzy cycle. Similarly the matrix of the second Hamiltonian fuzzy cycle is

$$\begin{matrix} V \\ E \end{matrix} \begin{pmatrix} 0.3 & 0.8 & 0.7 & 0.4 & 0.9 \\ 0.3 & 0.7 & 0.4 & 0.4 & 0.3 \end{pmatrix}$$

These two matrices have 2 rows and 5 columns. We cannot find the Eigen values for this 2x5 matrix. Therefore first we convert this matrix as a square matrix of 2x2 matrix using covariance of V and E. This converted matrix is called covariance matrix.

3.2 Covariance matrix and Eigen value of Hamiltonian fuzzy cycles

We find the values of cov (V, V), cov (V, E), cov (E, V) and cov (E, E) given below and tabulated as follows.

V	V	$V-V_0$	$V-V_0$	$(V-V_0)x(V-V_0)$
0.3	0.3	-0.32	-0.32	0.1024
0.4	0.4	-0.22	-0.22	0.0484
0.8	0.8	0.18	0.18	0.0324
0.9	0.9	0.28	0.28	0.0784
0.7	0.7	0.08	0.08	0.0064

V	E	$V-V_0$	$E-E_0$	$(V-V_0)x(E-E_0)$
0.3	0.3	-0.32	-0.2	0.064
0.4	0.4	-0.22	-0.1	0.022
0.8	0.8	0.18	0.3	0.054
0.9	0.7	0.28	0.2	0.056
0.7	0.3	0.08	-0.2	-0.016

E	E	$E-E_0$	$E-E_0$	$(E-E_0)x(E-E_0)$
0.3	0.3	-0.2	-0.2	0.04
0.4	0.4	-0.1	-0.1	0.01
0.8	0.8	0.3	0.3	0.09
0.7	0.7	0.2	0.2	0.04
0.3	0.3	-0.2	-0.2	0.04

$$\mathrm{Cov}(V, V) = \frac{\Sigma (V_i - V_0)(V_i - V_0)}{(n-1)}$$

$$\mathrm{Cov}(V, E) = \mathrm{Cov}(E, V) = \frac{\Sigma (V_i - V_0)(E_i - E_0)}{(n-1)}$$

$$\mathrm{Cov}(E, E) = \frac{\Sigma (E_i - E_0)(E_i - E_0)}{(n-1)}$$

Therefore we get cov (V, V) = 0.067, cov (V, E) = 0.045 = cov (V, E), cov (E, E) = 0.055 for the first cycle. Using the above values the 2x2 covariance matrix is founded as

$$C_1 = \begin{pmatrix} 0.067 & 0.045 \\ 0.045 & 0.055 \end{pmatrix}$$

The characteristic equation of the matrix is $|C_1 - \lambda_1 I| = 0$. Therefore we get the Eigen values of the first Hamiltonian fuzzy cycle as λ_1 = (0.10628, 0.01573). Similarly the covariance matrix of the second cycle is

$$C_2 = \begin{pmatrix} 0.067 & 0.0135 \\ 0.0135 & 0.027 \end{pmatrix}$$

and the Eigen value of this matrix is λ2 = (0.0711, 0.0229). From these two set of Eigen values the minimum value of the added Eigen value, indicates that corresponding fuzzy cycle has the shortest distance. Here λ2 have the shortest distance. This can be verified by simply adding the edges. Consider the membership values of vertices of K7 are (0.2 0.7 0.5 0.1 0.4 0.8 0.3). It can be decomposed that into 3 Hamiltonian fuzzy cycles, as follows

$$C_1 = \begin{matrix} V \\ E \end{matrix} \begin{pmatrix} 0.2 & 0.7 & 0.5 & 0.1 & 0.4 & 0.8 & 0.3 \\ 0.2 & 0.5 & 0.1 & 0.1 & 0.4 & 0.3 & 0.2 \end{pmatrix}$$

$$C_2 = \begin{matrix} V \\ E \end{matrix} \begin{pmatrix} 0.2 & 0.5 & 0.4 & 0.3 & 0.7 & 0.1 & 0.8 \\ 0.2 & 0.4 & 0.3 & 0.3 & 0.1 & 0.1 & 0.2 \end{pmatrix}$$

and

$$C_3 = \begin{matrix} V \\ E \end{matrix} \begin{pmatrix} 0.2 & 0.1 & 0.3 & 0.5 & 0.8 & 0.7 & 0.4 \\ 0.1 & 0.1 & 0.3 & 0.5 & 0.7 & 0.4 & 0.2 \end{pmatrix}$$

Using the above algorithm we get the Eigen values of the three matrices as λ_1 = (0.0754, 0.0132), λ_2 = (0.066, 0.0121) and λ_3 = (0.1093, 0.0054). Clearly the smallest added Eigen value lies on the second Hamiltonian fuzzy cycle. Thus C_2 has the shortest distance. Preceding like this it is possible to calculate the shortest distance Hamiltonian fuzzy cycle for all the complete fuzzy graphs of odd vertices.

3.3 Shortest distances measured in Hamiltonian fuzzy cycles using Eigen values of (2n) vertices

Here we are finding the shortest distance of a Hamiltonian fuzzy cycle from the Hamiltonian fuzzy decomposition of complete fuzzy graph with 2n vertices by using the above algorithm. If the fuzzy graph has 4 vertices then it has only one fuzzy cycle. Consider the complete fuzzy graph of six vertices with membership values (0.2 0.1 0.5 0.3 0.4 0.6). The matrix of two fuzzy cycles using the vertex and edge membership values, we get Using the above algorithm, the Eigen values of these matrices are λ_1 = (0.0385, 0.0102) and λ_2 = (0.0612, 0.0005).

$$\begin{matrix} V \\ E \end{matrix} \begin{pmatrix} 0.2 & 0.5 & 0.1 & 0.3 & 0.6 & 0.4 \\ 0.2 & 0.1 & 0.1 & 0.3 & 0.4 & 0.2 \end{pmatrix}$$

$$\begin{matrix} V \\ E \end{matrix} \begin{pmatrix} 0.2 & 0.1 & 0.6 & 0.5 & 0.4 & 0.3 \\ 0.1 & 0.1 & 0.5 & 0.4 & 0.3 & 0.2 \end{pmatrix}$$

Clearly λ_1 has the shortest distance. As an example consider the complete fuzzy graph with 8 vertices of membership values (0.1 0.5 0.7 0.3 0.2 0.1 0.6 0.3). Clearly this fuzzy graph can be decomposed into three Hamiltonian fuzzy cycles.

$$\begin{matrix} V \\ E \end{matrix} \begin{pmatrix} 0.1 & 0.5 & 0.3 & 0.7 & 0.6 & 0.3 & 0.1 & 0.2 \\ 0.1 & 0.3 & 0.3 & 0.6 & 0.3 & 0.1 & 0.1 & 0.1 \end{pmatrix}$$

$$\begin{matrix} V \\ E \end{matrix} \begin{pmatrix} 0.1 & 0.7 & 0.5 & 0.3 & 0.3 & 0.2 & 0.6 & 0.1 \\ 0.1 & 0.5 & 0.3 & 0.3 & 0.2 & 0.2 & 0.1 & 0.1 \end{pmatrix}$$

$$\begin{matrix} V \\ E \end{matrix} \begin{pmatrix} 0.1 & 0.3 & 0.7 & 0.2 & 0.5 & 0.1 & 0.3 & 0.6 \\ 0.1 & 0.3 & 0.2 & 0.2 & 0.1 & 0.1 & 0.3 & 0.1 \end{pmatrix}$$

The corresponding Eigen values are λ_1 = (0.0787, 0.0039), λ_2 = (0.0623, 0.0084) and λ_3 = (0.05143, 0.0078). The third cycle has the shortest distance.

Preceding like this it is possible to calculate the shortest distance Hamiltonian fuzzy cycle for all the complete fuzzy graphs of even vertices.

IV. CONCLUSION

From this work we can conclude that the shortest distance of Hamiltonian fuzzy decomposition of complete fuzzy graphs is possible to calculate by using Eigen values with covariance matrices.

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