International Journal of Current Trends in Engineering & Technology
ISSN: 2395-3152
Volume: 03, Issue: 05 (SEPTEMBER -OCTOBER, 2017)

Modified Bayes Thresholding based Wavelet Filter for Gaussian Noise Removal

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Abstract — *Wavelet based denoising is widely popular due to properties such as sparsity and multi-resolution structure. Wavelets based denoising attempts to remove noise present in the signal while preserving the signal characteristics, regardless of its frequency content. The wavelet based methods are based on the concept of thresholding the Discrete Wavelet transform coefficients affected by Gaussian noise. Wavelet-based denoising methods employ nonlinear thresholding of wavelet coefficients in the time-scale transform domain. There are a variety of thresholding techniques. This paper proposes a modified Bayes Thresholding based Wavelet Filter for Gaussian noise removal. The modified BayesShrink used is smoothness adaptive and sub band dependent which implies that thresholding is done at each band of resolution in the wavelet decomposition. The proposed algorithm is tested on different images and is found to produce better results in terms of the qualitative and quantitative measures of the image for both low and high density Gaussian noise images in comparison to many existing techniques.*

Keywords- *Gaussian noise, Wavelets, Bayes Thresholding.*

I. INTRODUCTION

The transmission and acquisition of images is frequently plagued by additive Gaussian noise. The Gaussian noise[1]-[10] is widely used to model thermal noise and, under some often reasonable conditions, is the limiting behavior of other types of noise, e.g., photon counting noise and film grain noise. Unlike impulse noise, which influences a few pixels only, this type of noise affects all pixel values.

Gaussian noise is characterized by adding to each image pixel a value from a zero-mean Gaussian distribution. The zero-mean property of the distribution helps in removing such noise by locally averaging pixel values.

A wide variety of linear and non-linear techniques[11]-[22] have been proposed in the literature including Median filter, Arithmetic filter, Gaussian Filter, Wiener Filter and the Wavelet transform approach. Conventional linear filters, such as arithmetic mean filter and Gaussian filter remove noise effectively but blur edges. The Wiener filter is the mean square error-optimal stationary linear filter for images degraded by additive noise and blurring. However a common drawback of the practical use of this method is the fact that they usually require some 'a priori' knowledge about the spectra of noise and the original signal. This information is essential for choosing the optimal values of both the parameter and the threshold values. Unfortunately, such information is often not available in real time applications. Also Wiener filter experiences uniform filtering throughout the image, with no allowance for changes between low and high frequency regions, resulting in unacceptable blurring of fine detail across edges and inadequate filtering of noise in relatively flat areas. Since the goal of the filtering action is to cancel noise while preserving the integrity of edge and detail information, nonlinear approaches generally provide more satisfactory results than linear techniques.

A number of techniques using wavelet-based thresholding [11], [13], [14], [18], [20]-[22] have been proposed recently by researchers. Wavelet transform, because of its signal representation with a high degree of sparseness and its excellent localization property, has rapidly become an indispensable image processing tool for a variety of applications, including denoising. A well known wavelet thresholding algorithm, named Wave Shrink, was introduced by Donoho in 1995 as a powerful tool for denoising signals degraded by additive white noise. Wave Shrink is based on the fact that for many of real-life signals, a limited number of wavelet coefficients in the lower sub bands are sufficient to reconstruct the original signal. Usually, the numerical values of these coefficients are relatively large as compared to noise coefficients. Therefore, by eliminating (shrinking) coefficients that are smaller than a specific value, called threshold, the noise can be nearly eliminated, while preserving the coefficients necessary to keep important attributes of the original image such as edges. Thus, choosing threshold values is extremely important. In the literature, various techniques for adaptive selection of threshold values, and new thresholding methods including fuzzy logic, neural networks, and wavelet packet base using Wiener filter are reported.

This paper proposes a modified Bayes Thresholding based Wavelet Filter where the Modified BayesShrink is smoothness adaptive and sub band dependent which implies that thresholding is done at each band of resolution in the wavelet decomposition. Computational simulation indicates that the proposed technique provides significant improvement over many other existing techniques in terms of PSNR for Gaussian noise removal.

The rest of the paper is organized as follows. The proposed methodology is introduced in Section II. The experimental results and comparison table are presented in Section III. The conclusions are provided in Section IV.

II. PROPOSED METHODOLOGY

Wavelet-based denoising is immensely prevalent due to properties such as sparsity and multi-resolution

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structure. Denoising by wavelet based transform is performed by standard thresholding algorithm which is governed mainly by either 'hard' or 'soft' thresholding function as shown below in Fig. 1.

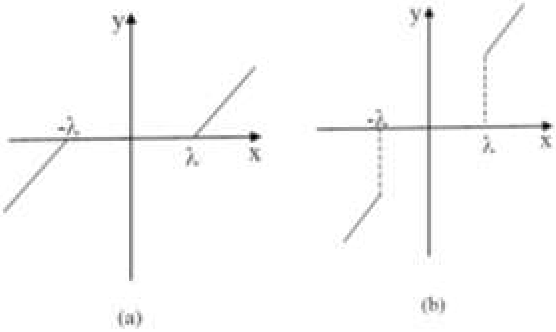

Fig. 1 Thresholding (a) Soft (b) Hard

In hard thresholding, the wavelet coefficients with values lying below the threshold λ are substituted by zero and coefficients above the threshold are not changed. If x, y denote the input and output respectively, then the hard threshold $T_h(y,\lambda)$ is given by:

$$T_h(y,\lambda) = \begin{cases} y, if\ |y| \geq \lambda \\ 0,\ otherwise \end{cases} \qquad (1)$$

In soft thresholding, the wavelet coefficients are shrunk towards zero by an offsetλ. The soft thresholding operator $T_s(y,\lambda)$ is given as

$$T_s(y,\lambda) = \begin{cases} y-\lambda, & if\ y \geq \lambda \\ y+\lambda, & if\ y \leq -\lambda \\ 0, & otherwise \end{cases} \qquad (2)$$

Generally, soft thresholding is preferred for the following reasons:

1) Soft-thresholding has been shown to achieve near-optimal minimax rate over a large range of Besov spaces.[7]
2) For the generalized Gaussian prior, the optimal soft-thresholding estimator yields a smaller risk than the optimum hard-thresholding estimator.
3) The soft-thresholding method yields more visually pleasant image over hard-thresholding because the

Latter is discontinuous and yields abrupt artifacts in the recovered images especially when the noise energy is significant.

Therefore, normally soft-threshold is used in most of the thresholding methods. The modified thresholding technique is now applied to the noisy image. The following three-stage approach is used to implement wavelets based thresholding methods to denoise the corrupted image.

Step 1: Computation of the discrete wavelet transform (DWT) of the noisy image.
Step 2: Removal of noise from the wavelet coefficients by employing suitable thresholding method.
Step 3: Reconstruction of the enhanced image using inverse discrete wavelet transforms (IDWT).

The modified BayesShrink uses soft thresholding and is sub band dependent [14], which means that thresholding is done at each band of resolution in the wavelet decomposition. The modified Bayes threshold (T_B) is smoothness adaptive and is given by:

$$T_B = \beta\ \frac{\sigma_n^2}{\sigma_s^2} \qquad (3)$$

Where σ_n^2 is the noise variance and σ_s^2 is the signal variance without noise. In the conventional Bayes threshold expression, $\beta = 1$. Here value of β is adaptive to different sub band characteristics and is given in eq.(4) as

$$\beta = log\ \frac{L}{k} \qquad (4)$$

Where L is the number of wavelet decomposition level and k is the level at which sub band is available.

The noise variance needs to be estimated first. In some situations, it may be possible to measure σ_n^2 based on information other than the corrupted image. If such is not the case, it is estimated from the subband HH_1 (fig. 2) by the robust mean estimator shown in eq.(5) below:

$$\sigma_n = \frac{Median(|Y_{ij}|)}{0.6745}, \quad Y_{ij} \in \text{sub band } HH_1 \qquad (5)$$

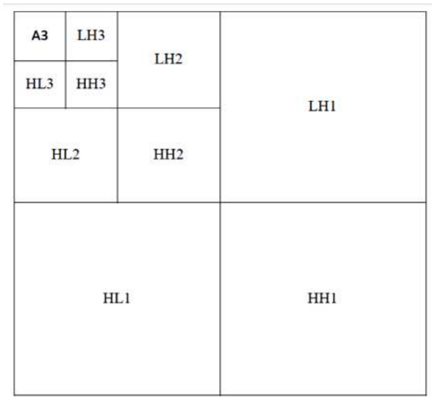

Fig. 2. Three levels of wavelet decomposition

From the definition of additive noise,

$$X(i,j) = f(i,j) + n(i,j) \qquad (6)$$

Where $X(i,j)$ is the (noise) corrupted image, $f(i,j)$ is the original signal and $n(i,j)$ is the noise. Since the noise and signal are independent of each other, it can be stated that

$$\sigma_x^2 = \sigma_f^2 + \sigma_n^2 \qquad (7)$$

Now σ_X^2 can be computed as follows:

$$\sigma_x^2 = \frac{1}{n^2}\sum_{i,j=1}^{n} X^2(i,j) \qquad (8)$$

The variance of the signal, σ_f^2, is computed as

$$\sigma_f^2 = \max(\sigma_X^2 - \sigma_n^2,\ 0) \qquad (9)$$

In the trivial case, the value of σ_f^2 may be zero, making the value of threshold T_B to be infinite. In this case, all the coefficients are set to zero. With σ_f^2 and σ_n^2, the Bayes threshold is computed using eq. (3). Using this expression of threshold, the wavelet coefficients are threshold at each sub band.

The process for reducing Gaussian noise using the Modified Bayes-Shrink wavelet based thresholding is described as follows:

Stage 1: Decomposition of the corrupted image with db8 wavelet at level 4 to obtain detail and approximate sub-band. In this case, there will be level 4 approximate sub-band, and four detail sub-bands each for level 1, 2, 3 and 4.

Stage 2: Apply the modified Bayes-Shrink soft threshold to each of the detail sub-bands so obtained to reduce noise.

Stage 3: Application of the inverse discrete wavelet transform to the output of stage 2 to obtain the denoised output.

III. SIMULATION RESULTS

This section compares the proposed algorithm with other existing techniques based on their simulation results. Peak signal-to-noise ratio (PSNR) is used to access the restoration results which measures how close the restored image is to the original image. The PSNR (dB) is defined as

$$\text{PSNR}= 10log_{10}\frac{(2^{b}-1)^{2}}{\frac{1}{MxN}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}(X(i,j)-Y(i,j))^{2}} \quad (10)$$

Where b refers to a b-bit image, M x N is the size of the image, *X(i,j)* refers to the original image and *Y(i,j)* refers to the denoised image. Since image is subjected to the human eyes, visual inspection is also carried out on the filtered images to judge the effectiveness of the filters in removing impulse noise.

TABLE 1

Noise (σ)→ / Filtering technique↓	5	10	15	20	25	30	35	40	45	50
INPUT PSNR	34.16	28.15	24.60	22.12	20.22	18.69	17.44	16.34	15.42	14.61
AWMDF (3x3)	34.88	32.58	30.44	28.58	27.01	25.67	24.52	23.49	22.53	21.65
AWMF (3x3)	34.03	32.80	31.33	29.85	28.52	27.25	26.22	25.23	24.39	23.59
WIENER FILTER	37.10	34.07	31.36	29.15	27.35	25.88	24.63	23.58	22.69	21.90
AVERAGE FILTER	35.9	31.3	28.1	25.5	23.64	22.5	21.43	20.2	19.73	18.4
VISU SHRINK	34.3	28.2	24.6	22.1	20.67	18.7	17.34	16.4	15.73	14.6
SURE SHRINK	25.1	25.1	25.1	25.1	25.1	25.0	24.9	24.8	24.7	24.6
MBTWF	37.94	34.06	31.96	30.60	29.61	28.83	28.19	27.46	26.85	26.39

Table 1 compares the restoration results in PSNR (dB) of the Modified Bayes Thresholding based Wavelet Filter (MBTWF) for 512x512 grayscale image 'Lena' corrupted by Gaussian noise of various noise levels. Fig. 3 compares the restoration results of various filters graphically, and is followed by their visual presentation in Fig. 4.

Comparison of restoration results of Modified Bayes-Shrink thresholding based Wavelet Filter in terms of PSNR(dB) for image 'Lena'

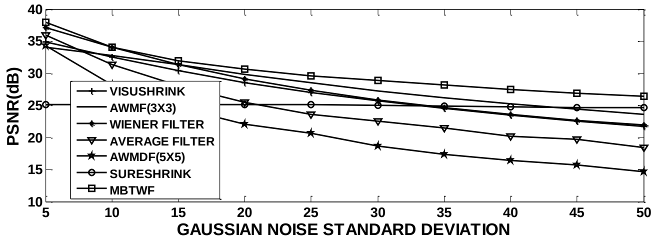

Fig. 3. Restoration Results for 'Len' Image

Fig. 4. Restoration Results for σ=50 for (a) AWMDF(3X3) b) AWMF(3X3) (c) Wiener Filter (d) Average Filter (e) VisuShrink (f) Sure Shrink (g) MBTWF

It can be seen from these figures that the proposed filter performs significantly better than the median, mean, Wiener, VisuShrink and Sure Shrink filter at both low and high noise levels at both quantitative and qualitative levels. It can reduce the Gaussian noise efficiently while preserving the edges at both low and high noise levels.

IV. CONCLUSION

A novel Modified Bayes Thresholding based Wavelet filter is proposed in this paper which can reduce Gaussian noise efficiently while preserving the edges very well. This technique is suitable for both low and high noise levels. The experimental results demonstrate that the proposed approach performs much better than other existing techniques in terms of both quantitative evaluation and visual quality.

Future research may be in the direction of reducing the processing time when the image is corrupted by high density of Gaussian noise.

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