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Two Storage Facilities Inventory Model for Defective Items with Different Rates of Deterioration and Linear Demand

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Abstract:- *Many times it happens that units produced or ordered are not of 100% good quality. A two storage facility inventory model for defective items with different deterioration rates is developed. Demand is considered as linear function of time. Holding cost is considered as function of time. Shortages are not allowed. To support the model, numerical example is given. Affectability investigation is likewise done for parameters.*

Keywords:- *Two storage facility, Different deterioration, Time dependent demand, defective Items.*

1. INTRODUCTION

The output of an item depends on many factors such as raw material quality, production process, organizational size, etc. Generally it is assumed that the items produced are 100% good quality items. Rosenblatt and Lee [10], Porteus [9] considered inventory model in which due to imperfect quality production process, the product produced are defective product. Salameh and Jaber [11] developed a model that defective items could be sold in a single batch at the end of 100% screening process. Chan et al. [2] developed inventory model in which products are classified as good quality, good quality after reworking, imperfect quality, and scrap. Sulak et al. [15] developed an EOQ model for defective items under permissible delay in payments and shortages. Many time retailers decide to buy goods exceeding their Own Warehouse (OW) capacity to take advantage of price discounts. Therefore an additional stock is arranged as Rented Warehouse (RW) which has better storage facilities with higher inventory holding cost. Hartley [3] first developed a two warehouse inventory model. Sharma [13] developed an inventory model with finite rate of replenishment with two warehouses. Pakkala and Achary [7] developed a deterministic inventory model for deteriorating items with two warehouses and finite replenishment rates. Bhunia [1] proposed two warehouse inventory model for deteriorating items with linear demand and shortages. Lee and Hsu [6] considered a two warehouse inventory model with time dependent demand. Sana et al. [12] proposed a two warehouse inventory model on pricing decision. Yu [17] gave two warehouse inventory model for deteriorating items with decreasing rental over time. Tyagi [16] proposed model with time dependent and variable holding cost. Parekh and Patel [8] developed deteriorating item inventory models for two warehouses with linear demand under inflation and permissible delay in payments. Sheikh and Patel [14] developed a two warehouse inventory model under linear demand and time varying holding cost. Jaggi et al. [4] formulated a two warehouse deteriorating items inventory model under price dependent demand. Jaggi et al. [5] considered a two warehouse inventory model for imperfect quality items. Generally the products are such that there is no deterioration initially. After certain time deterioration starts and again after certain time the rate of deterioration increases with time. Here we have used such a concept and developed the deteriorating items inventory model. In this paper we have developed a two warehouse defective items inventory model with different deterioration rates. Demand function and holding cost is time dependent. Shortages are not allowed. Numerical case is given to represent the model. Affectability investigation is likewise done for parameters.

2. ASSUMPTIONS AND NOTATIONS

NOTATIONS

The following notations are used for the development of the model

(t)	:	Demand is a function of time (a + bt, a>0, 0<b<1)
HC(OW)	:	Holding cost is function of time t (x1+y1t, x1>0, 0<y1<1) in OW.
HC(RW)	:	Holding cost is function of time t (x2+y2t, x2>0, 0<y2<1) in RW.
A	:	Ordering cost per order
c	:	Purchasing cost per unit
p	:	Selling price per unit
d	:	Defective items (%)
1-d	:	Good items (%)
λ	:	Screening rate
SR	:	Sales revenue
z	:	Screening cost per unit
p_d	:	Price of defective items per unit
t_1	:	Screening time
T	:	Length of inventory cycle
$I_0(t)$	:	Inventory level in OW at time t
$I_r(t)$	:	Inventory level in RW at time t
Q	:	Order quantity
t_r	:	Time at which inventory level becomes zero in RW.
W	:	Capacity of own warehouse
θ	:	Deterioration rate in OW during μ1<t< μ2, 0< θ<1
θt	:	Deterioration rate in OW during μ2 ≤ t ≤ T, 0< θ<1
π	:	Total relevant profit per unit time.

ASSUMPTIONS

The following assumptions are considered for the development of model. The demand of the product is declining as a function of time. Replenishment rate is infinite and instantaneous. Lead time is zero. Shortages are not allowed. The screening process and demand proceeds simultaneously but screening rate (λ) is greater than the demand rate i.e. $\lambda > (a+bt)$. The defective items are independent of deterioration. Deteriorated units can neither be repaired nor replaced during the cycle time. A single product is considered. Holding cost is time dependent. The screening rate (λ) is sufficiently large. In general, this assumption should be acceptable since the automatic screening machine usually takes only little time to inspect all items produced or purchased. OW has fixed capacity W units and RW has unlimited capacity. The goods of OW are consumed only after consuming the goods kept in RW. The unit inventory cost per unit in the RW is higher than those in the OW.

3. THE MATHEMATICAL MODEL AND ANALYSIS:

In the following situation, Q items are received at the beginning of the period. Each lot having a d % defective items. The nature of the inventory level is shown in the given figure, where screening process is done for all the received items at the rate of λ units per unit time which is greater than demand rate for the time period 0 to t_1. d% of defective items are separated and from remaining Q – dQ = Q(1-d) good items, W units are stored in own warehouse (OW) and remaining Q(1-d)-W units are stored in rented warehouse(RW).

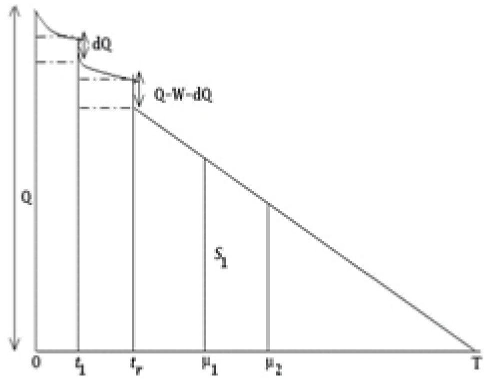

Figure 1

During the screening process the demand occurs parallel to the screening process and is fulfilled from the goods which are found to be of perfect quality by screening process in rented warehouse. The defective items are sold immediately after the screening process at time t_1 as a single batch at a discounted price. After the screening process at time t_1 the inventory level will be $I(t_1)$. At time t_r level of inventory in RW reaches to zero because of demand and OW inventory remains W. During the interval (t_r,μ_1) inventory depletes in OW due to demand, during interval (μ_1, μ_2) inventory depletes in OW due to deterioration at rate θ and demand. During interval (μ_2, T) inventory in OW depletes due to joint effect of deterioration at rate θt and demand. By time T both the warehouses are empty.

Also here $$t_1 = \frac{Q}{\lambda} \qquad (1)$$

and defective percentage (d) is restricted to

$$d \le 1 - \frac{(a+bt)}{\lambda} \qquad (2)$$

Let I(t) be the inventory at time t $(0 \le t \le T)$ as shown in figure. 1. Hence, the inventory level at time t in RW and OW and governed by the following differential equations:

$$\frac{dI_r(t)}{dt} = -(a+bt), \qquad 0 \le t \le t_r \qquad (3)$$

$$\frac{dI_0(t)}{dt} = 0, \qquad 0 \le t \le t_r \qquad (4)$$

$$\frac{dI_0(t)}{dt} = -(a+bt), \qquad t_r \le t \le \mu_1 \qquad (5)$$

$$\frac{dI_0(t)}{dt} + \theta I_0(t) = -(a+bt), \qquad \mu_1 \le t \le \mu_2 \qquad (6)$$

$$\frac{dI_0(t)}{dt} + \theta t I_0(t) = -(a+bt), \qquad \mu_2 \le t \le T \qquad (7)$$

with initial conditions $I_0(0) = W$, $I_0(\mu_1) = S_1$, $I_0(t_r) = W$, $I_r(0) = Q(1-d)-W$, $I_r(t_r) = 0$ and $I_0(T)=0$.

Solving equations (3) to (7) we have,

$$I_r(t) = Q - W - at - \frac{1}{2}bt^2 \qquad (8)$$

$$I_0(t) = W \qquad (9)$$

$$I_0(t) = S_1 + a(\mu_1 - t) + \frac{1}{2}b(\mu_1^2 - t^2) \qquad (10)$$

$$I_0(t) = \begin{bmatrix} a(\mu_1 - t) + \frac{1}{2}b(\mu_1^2 - t^2) + \frac{1}{2}a\theta(\mu_1^2 - t^2) \\ + \frac{1}{3}b\theta(\mu_1^3 - t^3) - a\theta t(\mu_1 - t) - \frac{1}{2}b\theta t(\mu_1^2 - t^2) \end{bmatrix} + S_1(1 + \theta(\mu_1 - t)) \qquad (11)$$

$$I_0(t) = \begin{bmatrix} a(T-t) + \frac{1}{2}b(T^2 - t^2) + \frac{1}{6}a\theta(T^3 - t^3) \\ + \frac{1}{8}b\theta(T^4 - t^4) - \frac{1}{2}a\theta t^2(T-t) - \frac{1}{4}b\theta t^2(T^2 - t^2) \end{bmatrix} \qquad (12)$$

(by neglecting higher powers of θ) After screening process, the number of defective items at time t_1 is dQ. So effective inventory level during $t_1 \le t \le T$ is given by

$$I_r(t) = Q(1-d) - W - at - \frac{1}{2}bt^2. \qquad (13)$$

Putting t = tr in equation (13), we get

$$Q = \frac{1}{(1-d)}\left[W + at_r + \frac{1}{2}bt_r^2\right] \qquad (14)$$

Putting t = tr in equations (9) and (10), we get

$$I_0(t_r) = W \qquad (15)$$

$$I_0(t_r) = S_1 + a(\mu_1 - t_r) + \frac{1}{2}b(\mu_1^2 - t_r^2) \qquad (16)$$

So from equations (15) and (16), we have

$$S_1 = W - a(\mu_1 - t_r) - \frac{1}{2}b(\mu_1^2 - t_r^2) \qquad (17)$$

Putting t = μ_2 in equations (11) and (12), we get

$$I_0(t) = \begin{bmatrix} a(\mu_1 - \mu_2) + \frac{1}{2}b(\mu_1^2 - \mu_2^2) \\ + \frac{1}{2}a\theta(\mu_1^2 - \mu_2^2) + \frac{1}{2}b(\mu_1^2 - \mu_2^2) \\ + \frac{1}{3}b\theta(\mu_1^3 - \mu_2^3) - a\theta t(\mu_1 - \mu_2) \\ - \frac{1}{2}b\theta t(\mu_1^2 - \mu_2^2) \end{bmatrix} + S_1(1 + \theta(\mu_1 - \mu_2)) \qquad (18)$$

$$I_0(t) = \begin{bmatrix} a(T - \mu_2) + \frac{1}{2}b(T^2 - \mu_2^2) \\ + \frac{1}{6}a\theta(T^3 - \mu_2^3) + \frac{1}{8}b\theta(T^4 - \mu_2^4) \\ - \frac{1}{2}a\theta\mu_2^2(T - \mu_2) - \frac{1}{4}b\theta t^2(T^2 - \mu_2^2) \end{bmatrix} \qquad (19)$$

So from equations (15) and (16), we have

$$T = \frac{1}{b(\theta\mu_2^2 - 2)}\left(\begin{array}{l} 2a - a\theta\mu_2^2 \\ + \sqrt{\begin{array}{l} 4b\theta\mu_2^2 a\mu_1 - 8ab\theta t_r\mu_2 + 8ab\theta t_r\mu_1 \\ -4b\theta^2\mu_2^2 at_r\mu_1 - 4b\theta\mu_2^2 at_r - 4b\theta^2\mu_2^2 W\mu_1 \\ + 4b\theta^2\mu_2^2 a\mu_1^2 - 4b\theta^2\mu_2^3 a\mu_1 + 4b\theta^2\mu_2^3 at_r \\ - 4a^2\theta\mu_2^2 + a^2\theta^2\mu_2^4 - 8ab\mu_1 + 8abt_r \\ + 8ab\mu_2 - 2b^2\theta\mu_2^4 + 8ab\theta\mu_1\mu_2 \\ + 4b^2\mu_2^2 + 4a^2 + 8bW - 4b^2\mu_1^2 \\ -4b^2t_r^2 + 8bW\theta\mu_1 - 8bW\theta\mu_2 - 8ab\theta\mu_1^2 \\ + 4b^2\theta\mu_2\mu_1^2 - 4b^2\theta\mu_2 t_r^2 - 4bW\theta\mu_2^2 \\ + 2b^2\theta\mu_2^3\mu_1^2 - 2b^2\theta\mu_2^2 t_r^2 + 4bW\theta^2\mu_2^3 \\ - 2b^2\theta^2\mu_2^3\mu_1^2 + 2b^2\theta^2\mu_2^3 t_r^2 - 4ab\theta\mu_2^3 \end{array}} \end{array} \right) \qquad (20)$$

From equation (20), we see that T is a function of W and t_r, so T is not a decision variable. Based on the assumptions and descriptions of the model, the total annual relevant profit(π), include the following elements:

(i) Ordering cost (OC) = A (21)

$$\text{(ii) } HC(OW) = \int_0^{t_r}(x_1 + y_1 t)I_0(t)dt + \int_{t_r}^{\mu_1}(x_1 + y_1 t)I_0(t)dt + \int_{\mu_1}^{\mu_2}(x_1 + y_1 t)I_0(t)dt + \int_{\mu_2}^{T}(x_1 + y_1 t)I_0(t)dt \qquad (22)$$

$$\text{(iii) } HC(RW) = \int_0^{t_1}(x_2 + y_2 t)I_r(t)dt + \int_{t_1}^{t_r}(x_2 + y_2 t)I_r(t)dt \qquad (23)$$

$$\text{(iv) } DC = c\left(\int_{\mu_1}^{\mu_2}\theta I_0(t)\,dt + \int_{\mu_2}^{T}\theta t I_0(t)\,dt\right) \qquad (24)$$

(v) SR = Sum of sales revenue generated by demand meet during the period (0,T) + Sales of imperfect quality items

$$= \left(p\int_0^T (a + bt)dt + p_d dQ\right)$$

$$= p\left(aT + \frac{1}{2}bT^2\right) + p_d dQ \qquad (25)$$

(by neglecting higher powers of θ)

The total profit (π) during a cycle consisted of the following:

$$\pi = \frac{1}{T}\left[SR - OC - SrC - HC(RW) - HC(OW) - DC\right] \qquad (26)$$

Substituting values from equations (21) to (25) in equation (26), we get total profit per unit. Putting $\mu_1 = v_1 T$, $\mu_2 = v_2 T$ and value of S_1 and T from equation (17) and (20) in equation (26), we get profit in terms of tr. The optimal value of t_r* (say), which maximizes π can be obtained by solving equation (26) by differentiating it with respect to t_r and equate it to zero

i.e. $\frac{d\pi}{dT} = 0$ (27)

provided it satisfies the condition

$$\frac{d^2\pi}{dT^2} < 0. \qquad (28)$$

4. NUMERICAL EXAMPLE

Considering A= Rs.100, W = 136, a = 500, b=0.05, c=Rs. 25, p= Rs. 25, d=0.02, p_d= Rs. 15, λ=10000, θ=0.05, x_1 = Rs. 3, y_1=0.05, x_2 = Rs. 6, y_2=0.06, v_1=0.30, v_2=0.50, in appropriate units. The optimal value of t_r*=0.0385, Profit*=Rs.19359.9946 and Q*=158.4184. The second order condition given in equation (28) is also satisfied. The graphical representation of the concavity of the profit function is also given.

5. SENSITIVITY ANALYSIS

On the basis of the data given in example above we have studied the sensitivity analysis by changing the following parameters one at a time and keeping the rest fixed. From the table we observe that as parameter a increases/ decreases average total profit and order quantity increases/ decreases. From the table we observe that as parameter θ increases/ decreases there is very minor change in average total profit and order quantity. From the table we observe that as parameters x_1 and x_2, increase/ decrease average total profit and order quantity decrease/ increase.

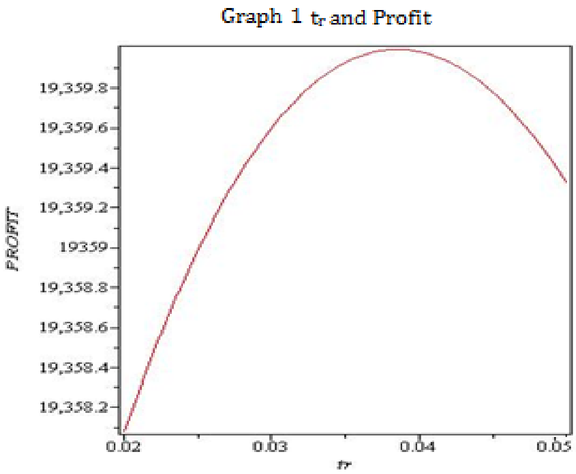

Table 1 Sensitivity Analysis

Parameter	%	t_r	Profit	Q
a	+20%	0.0485	23288.0549	168.4694
	+10%	0.0441	21323.5737	163.5255
	-10%	0.0313	17397.4004	153.1480
	-20%	0.0218	15435.8853	147.6735
θ	+20%	0.0365	19354.1397	157.3980
	+10%	0.0375	19357.0615	157.9082
	-10%	0.0395	19362.9392	158.9286
	-20%	0.0406	19365.8953	159.4898
x_1	+20%	0.0273	19314.8004	152.7041
	+10%	0.0330	19337.2315	155.6123
	-10%	0.0439	19383.0720	161.1735
	-20%	0.0492	19406.4474	163.8776
x_2	+20%	0.0328	19358.5672	155.5102
	+10%	0.0354	19359.2338	156.8367
	-10%	0.0422	19360.8750	160.3061
	-20%	0.0466	19361.9097	162.5511
A	+20%	0.0576	19297.3077	168.1633
	+10%	0.0482	19328.1847	163.3674
	-10%	0.0285	19392.8290	153.3163
	-20%	0.0181	19426.7953	148.0102
λ	+20%	0.0385	19360.1568	158.4184
	+10%	0.0385	19360.0831	158.4184
	-10%	0.0385	19359.8865	158.4184
	-20%	0.0384	19359.7514	158.3673

From the table we observe that as parameter A increases/ decreases average total profit decreases/ increases and order quantity increases/ decreases. From the table we observe that as parameter λ increases/ decreases there is almost no change in average total profit and order quantity.

6. PARTICULAR CASE:

When p_d=0, d=0, we get tr*=0.0388, Profit=Rs. 19412.1471, which is same as Sheikh and Patel [14].

7. CONCLUSION

In this paper, we have developed a two warehouse defective items inventory model for deteriorating items with different deterioration rates under time dependent demand, and time varying holding cost. Sensitivity with respect to parameters has been carried out. The results show that with the increase/ decrease in the parameter values there is corresponding increase/ decrease in the value of profit.

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