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Oxide Tiling System and Oxide Wang System

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Abstract—*In this paper we introduce Oxide Wang tiles, local Oxide picture languages and recognizable Oxide picture languages. We define local oxide picture languages and its recognizability by Oxide tiling system. We also introduce Oxide Wang systems (OWS), a formalism to recognize Oxide picture languages. Comparative results of Oxide Wang System with Oxide Tiling system have also been explored here.*

Keyword—Oxide picture languages, local and recognizable Oxide languages, Oxide tiling systems, Oxide Wang systems.

Introduction

Pictures are described as digitized finite arrays in a rectangular grid. Picture languages are generated by grammars or recognized by automata [3-7]. Tiling systems were introduced in [1] [5] defining local and recognizable picture languages. In each picture of the language a specified set of square tiles is required to define local picture language. On the other hand in [8] Wang system is used as a formalism to recognize picture languages. Pictures recognized by Wang systems are proved to be equivalent to recognizability of pictures by Tiling System. We require certain tiles "Star of David tiles" only to be present in each Oxide picture of a local Oxide picture language. This leads on to the notion of Oxide tiling system defining recognizable Oxide picture languages. We also define Oxide Wang tiles and Oxide Wang systems. In section 3 we define Oxide tiling system, local Oxide tiling languages and recognizable Oxide tiling languages. In section 4 we show the recognizability of Oxide tiling languages by Wang systems. Studies have been done on Oxide tiling languages and results of Oxide tiling system with Oxide Wang system have also been compared.

Preliminaries

The silicates are the most complicated and the most interesting class of minerals by far. Tetrahedron (SiO_4) is the basic chemical unit of silicates. In a two dimensional plane a silicate sheet (Fig. 1) is a ring of tetrahedrons which are linked by shared oxygen nodes to other rings that produces a sheet-like structure. A silicate network is a parallel fixed interconnection of silicate sheets. Place silicon ions on all the vertices of HC (n). Subdivide once each edge of HC (n). Oxygen ions are placed on the new vertices. At the 2-degree silicon ions of HC (n) introduce $6n$ new pendant edges one each and at the pendent vertices place oxygen ions. See Fig. 3(a). Associate the three adjacent oxygen ions with every silicon ion and form a tetrahedron Fig. 3(b). By successively fusing oxygen nodes of two tetrahedra of different silicates the minerals are obtained. A silicate network can be constructed in different ways [10]. From a honeycomb network we describe the construction of a silicate network. A honeycomb network can be built from a hexagon in various ways [14, 15].

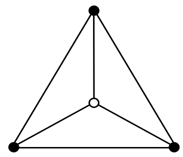

Fig. 1. SiO4 tetrahedra where the corner vertices represent oxygen ions and the center vertex the silicon ion

The honeycomb network HC (1) is a hexagon. The honeycomb network HC (2) is obtained by adding six hexagons to the boundary edges of HC (1). Inductively, honeycomb network $HC(n)$ is obtained from $HC(n-1)$ by adding a layer of hexagons around the boundary of $HC(n-1)$. Fig. 2 is HC (3). The dimension of HC (n) is the parameter n

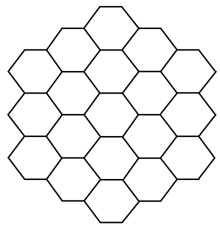

Fig. 2. A honeycomb network $HC(3)$

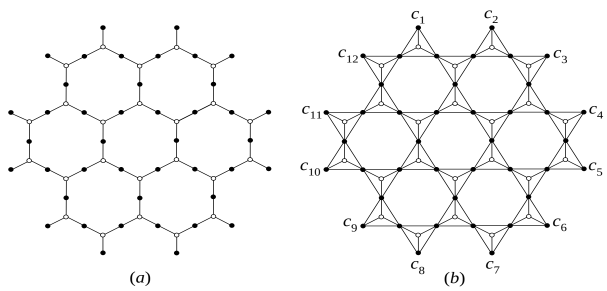

Fig. 3. Silicate network construction and boundary nodes

The resulting network is a silicate network SL (n) of dimension n. The diameter of SL (n) is $4n$. The graph in Fig. 3(b) is a silicate network of dimension two. The number of nodes in SL (n) is $15n^2 + 3n$ and the number of edges of SL (n) is $36n^2$ [10]. A new network which we shall call as an *Oxide Network* is obtained by deleting all the silicon nodes from a silicate network (Fig. 4). An n-dimensional oxide network is denoted by OX (n).The number of nodes in OX (n) is $9n^2 + 3n$ and edges $18n^2$ [10].

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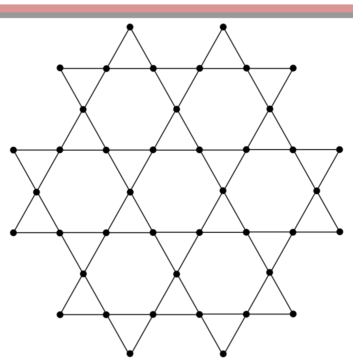


Fig. 4. An Oxide Network OX (2)

For Oxide network a coordinate system is proposed that assigns an *id* to each Oxygen node. Nocetti et al. [9] and Stojmenovic [14] proposed a coordinate system for a hexagonal network and a honeycomb network respectively.

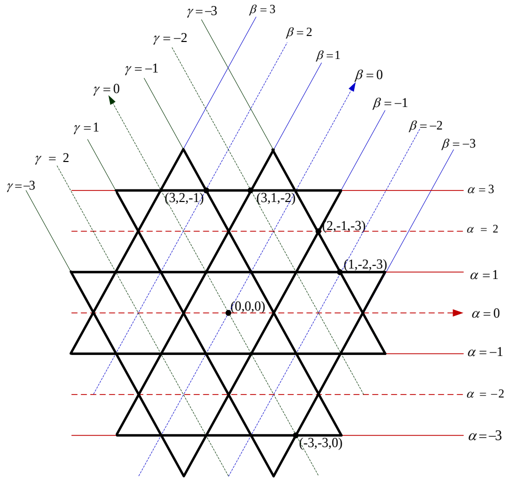


Fig. 5. Coordinate System in Oxide Networks

At mutual angle of 120 degrees between any two of them α, β and γ are the three axes parallel to three edge directions. The three coordinate axes are $\alpha = 0$, $\beta = 0$, and $\gamma = 0$ respectively and α-lines, β-lines, γ-lines are the lines parallel to the coordinate axes. Here $\alpha = h$ and $\alpha = -k$ are α-lines on either side of α - axis. A pixel of OX (n) is assigned a triple (a, b, c) when the node is the intersection of lines $\alpha = a$, $\beta = b$, and $\gamma = c$, $a+b+c+2 \equiv 0 \bmod 4$. Each silicon node is assigned *ids* at the centroid of three oxygen nodes of a tetrahedral SiO_4 by applying the formula of centroid of an equilateral triangle (Fig. 5).

Oxide Pictures and Oxide Picture Languages

In this section we introduce Oxide pictures, Oxide picture languages and also generalize the concept of formal language theory relating to rectangular pictures to Oxide pictures and define Oxide picture languages.

Definition 3.1: Let Σ be a finite alphabet. An Oxide picture OX_p over Σ is an oxide array of symbols of Σ. The set of all Oxide pictures over the alphabet Σ is denoted by Σ^{**OX_p}. An Oxide picture language over Σ is a subset of Σ^{**OX_p}. OX_p (n) denotes Oxide picture of size n.

Example 1. An Oxide picture over the alphabet {a, b, c} is shown in Fig. 6

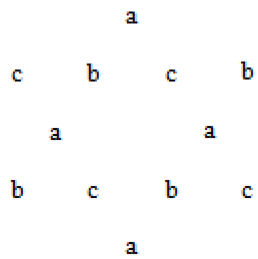


Fig. 6. An Oxide Picture

Definition 3.2: Let Σ and Γ be two finite alphabets and $OX_p \in \Gamma^{**OX_p}$ be an Oxide picture. The projection by a mapping π of OX_p is the Oxide picture $OX_p{}' \in \Sigma^{**OX_p}$ such that $OX_p{}'(a,b,c) = \pi(OX_p(a,b,c))$

Definition 3.3: Let $L \subseteq \Gamma^{**OX_p}$ be an Oxide picture language. The projection by a mapping, π of L is the language

$$L' = \{OX_p{}' \,/\, OX_p{}' = \pi(OX_p), \forall OX_p \in L\} \subseteq \Sigma^{**OX_p}$$

Definition 3.4: Let $OX_p \in \Sigma^{**OX_p}$ we get a bordered version of OX_p say $\hat{OX}_p$ when $\# \notin \Sigma$ is added as boundary to the Oxide picture OX_p.

Recognizability of Oxide Picture by Oxide Tiling System

4.1. The Family OXLOC

In this section, the main notions of local and recognizable Oxide picture languages are introduced.
We define local Oxide picture languages by means of a set of tiles (Fig. 7 - Star of David tile $OX_p(1)$) that represent the only allowed blocks of that size in pictures of the Oxide picture language. Then we say that an Oxide tiling language is "tiling recognizable" if it can be obtained as a projection of a local Oxide picture language.

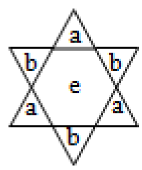
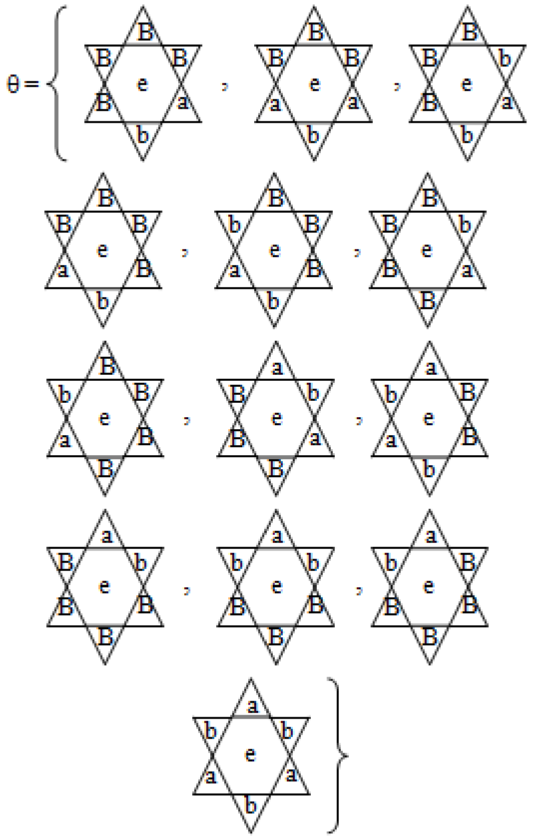


Fig. 7: An Oxide tile $OX_p(1)$ – Star of David tile

Definition 4.1: Let Γ be a finite alphabet. An Oxide picture language $L \subseteq \Gamma^{**OX_p}$ is local if there exists a finite set of Star of David tiles over $\Gamma \cup \{\#\}$ such that $L = \{OX_p \in \Gamma^{**OX_p} / OX_p(1) \subseteq \theta\}, \theta$ represents the set of all allowed blocks for pictures belonging to the local Oxide language L.

Example 4.1 Let $\Gamma = \{a, b\}$ be an alphabet and let θ be a set of Star of David tiles over Γ



The language L = L(θ) is the language of oxide pictures in which all A triangle carry symbol "a" and V triangle carry symbol "b" . The family of local Oxide picture languages will be denoted by OXLOC. To understand the notion of local oxide picture language in terms of a computational procedure to recognize an Oxide picture a Star of David tile is moved around the Oxide picture and a record is made of the blocks observed through the window. The number of occurrences these blocks and the order is regardless. An Oxide picture language "is accepted "if the set of the recorded blocks is included in the given set of Star of David tiles.

4.2 The Family OXREC

We define the family of recognizable Oxide picture languages using the notion of local Oxide picture languages introduced above and the notion of projection of an Oxide Picture language. Combining these two, yields the definition of an Oxide tiling system.

Definition 4.2: Let Σ be a finite alphabet. An Oxide picture language $L \subseteq \Sigma^{**OX_p}$ is called recognizable if there exists an Oxide local picture language L' (given by a set of Star of David tiles) over an alphabet Γ and a mapping $\pi : \Gamma \to \Sigma$ such that $L = \pi(L')$. The family of all recognizable oxide picture languages will be denoted by OXREC.

Definition 4.3: An oxide tiling system $OXTS$ is a 4 tuple $(\Sigma, \Gamma, \pi, \theta)$ Where Σ and Γ are two finite alphabets and the mapping $\pi : \Gamma \to \Sigma$ is a projection and θ is a set of Star of David tiles over the alphabet $\Gamma \cup \{\#\}$

Definition 4.4: An oxide tiling Language $L \subseteq \Gamma^{**OX_p}$ is tiling recognizable if there exists an oxide tiling system $OXTS = (\Sigma, \Gamma, \pi, \theta)$ such that $L = \pi(L(\theta))$. OXREC is exactly the family of oxide tiling languages recognizable by oxide tiling system (OXTS).

Theorem 4.1: The family L (OXTS) is closed under projection

Proof: Let $T_1 = (\Sigma_1, \Gamma, \pi_1, \theta)$ be a tiling system for L_1 .we define $T_2 = (\Sigma_2, \Gamma, \pi_2, \theta)$ be some Oxide tiling system and let $\varphi : \Sigma_1 \to \Sigma_2$ be a projection. We have to prove that, if $L_1 \subseteq \Sigma_1^{**OX_p}$ is recognizable by an Oxide tiling system then $L_2 = \varphi(L_1)$ is recognizable by an Oxide tiling system too. As $T_1 = (\Sigma_1, \Gamma, \pi_1, \theta)$ is a tiling system for L_1 there is an underlying local Oxide picture language L' such that $L_1 = \pi_1(L')$. We prove that L' is an underlying local oxide picture language also for L_2. For this purpose set $\pi_2 = \varphi.\pi_1$, π_2 is a projection from Γ to Σ_2. Then L' is also an underlying local Oxide picture language also for L_2.

Recognizability of Oxide Pictures by Oxide Wang System

In this section we introduce Oxide Wang tiles to define Oxide Wang system (OXWS), a formalism to recognize Oxide picture language.

Definition 5.1: A labeled Oxide Wang tile is a 7 tuple consisting of labeled triangular Wang tile (A tile and V tile) of three colors from a finite set of colors $Q_1 \subset Q_2$, a labeled hexagonal Wang tile consisting of 6 colors from a finite set of colors Q_2 and a label from a finite alphabet Σ . The colors are placed at left (L) , right (R) and horizontal (H) positions of the labeled triangular Wang tile and at the upper horizontal (UH) , lower horizontal (LH) ,upper left (UL) , upper right (UR), lower left (LL), lower right (LR) positions of the hexagonal Wang tile (fig. 8).

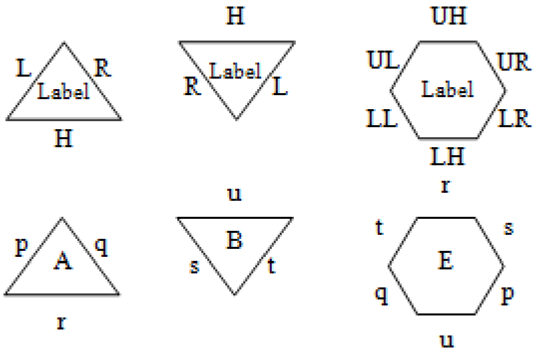
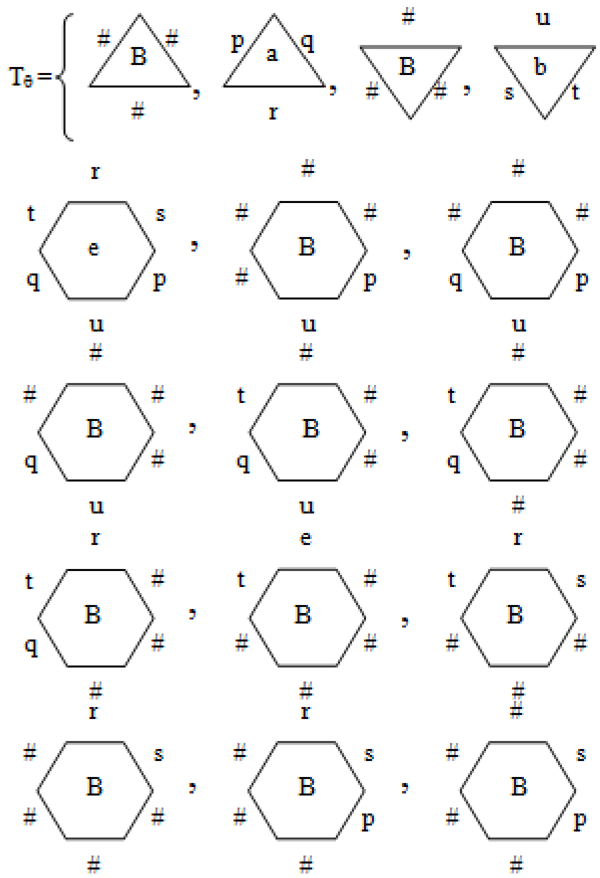


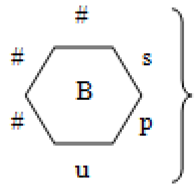
Fig. 8. Triangular and Hexagonal Wang tiles

A triangular Wang tile may be adjacent with a hexagonal Wang tile if the adjacent edges are of the same color and no two triangular tiles are adjacent.

Definition 5.2: An Oxide Wang system is a triplet (Σ, Q, T_θ) where Σ is a finite alphabet, Q is a finite set of colors, T_θ is a set of labeled Oxide Wang tiles. The language generated by $OXWS$ is the language $L(OXWS) \subseteq \Sigma^{**OX(n)}$ of the labels of an Oxide Wang pictures is $L_w(T_\theta)$. OXWREC is the class of picture language generated by Oxide Wang system.

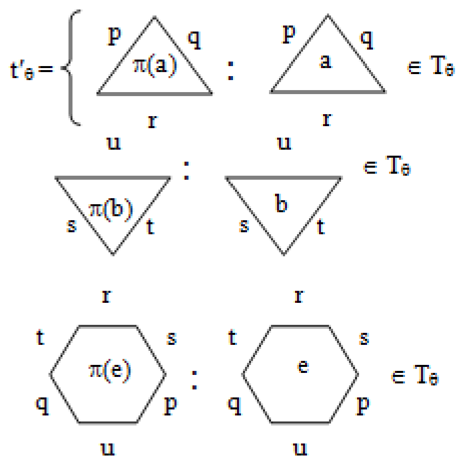
Example5.1 Consider the language $L \subseteq \Sigma^{**OX(p)}$ of picture of size $n \geq 2$ with the Type A triangle carry letter 'a' and type V triangle carry letter 'b'. Then L is recognized by the Oxide Wang system (Σ, Q, T_θ) where $Q = \Sigma \cup \{\#, p, q, r, s, t, u\}$ and



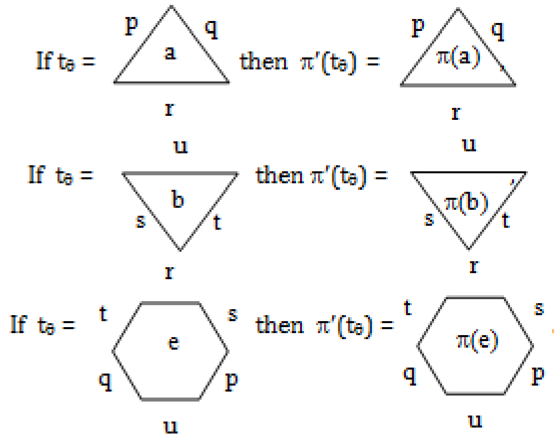


Theorem 5.1: $L(OXWS)$ is closed under projection

Let $W = (\Sigma_1, Q, T_\theta)$ be an Oxide Wang system. We define $W' = (\Sigma_2, Q, T_\theta')$ be some Oxide Wang system where $\Pi : \Sigma_1 \to \Sigma_2$ be a projection. We prove that $L' = \Pi(L(W))$ is Oxide Wang recognizable. That is $L' = L(W')$ for some Oxide Wang system W' . Define

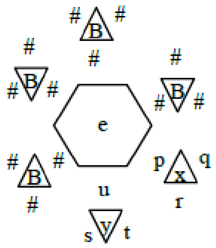
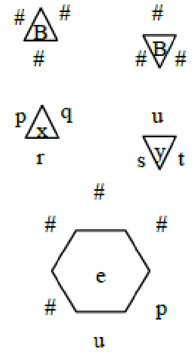


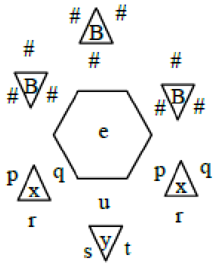
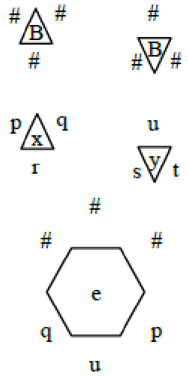
Let $\pi' : T_\theta \to T'_\theta$ be the projection defined by

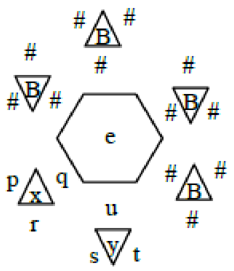
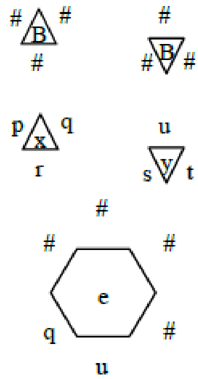


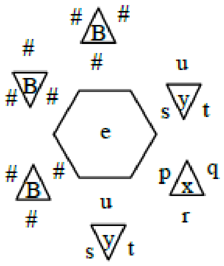
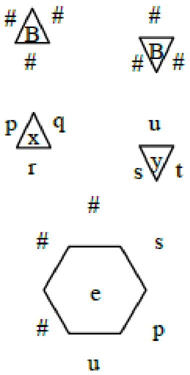
Let $w \in L(W)$ then there exists an Oxide tiling H of W whose label is w and $\pi'(H)$ is an Oxide tiling of W' with label $w' = \pi(w)$. Conversely if $W' = \pi(W) \in L(W')$ then w' is the label of an Oxide tiling H' of W' and $H' = \pi'(H)$ where H is an Oxide tiling of W whose label is w.

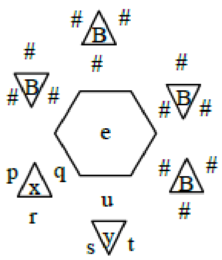
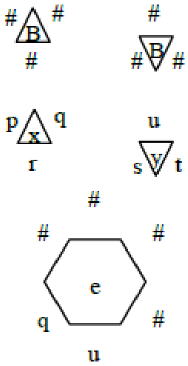
Theorem5. 2: $L(OXWS) \subseteq L(OXTS)$. Let $L \in L(OXWS)$ then $L = L(W)$ and $W = (\Sigma_1, Q, T_\theta)$. We find an Oxide tiling system $\tau = (\Sigma_2, \Gamma, \theta, \pi)$ such that $L = L(W) = L(\tau)$ For this purpose construct Star of David tile θ_i using tiles of T_θ as follows

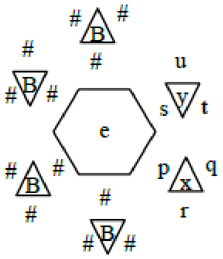
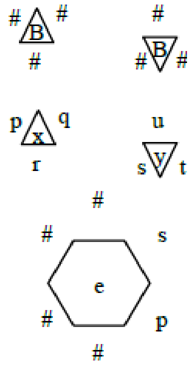
$\theta_1 =$  $: T_{\theta_1} =$ 

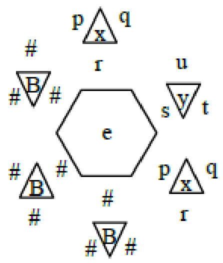
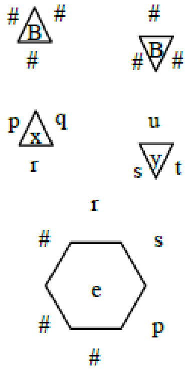
$\theta_2 =$  $: T_{\theta_2} =$ 

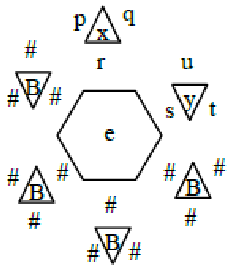
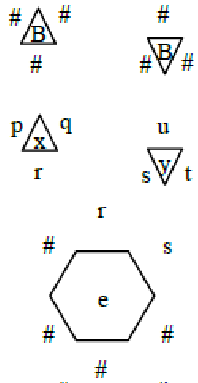
$\theta_3 =$  $: T_{\theta_3} =$ 

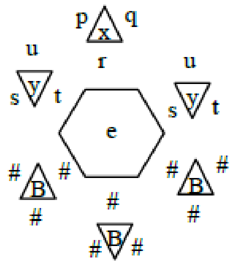
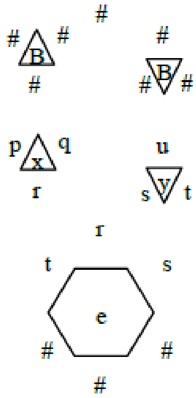
$\theta_4 =$  $: T_{\theta_4} =$ 

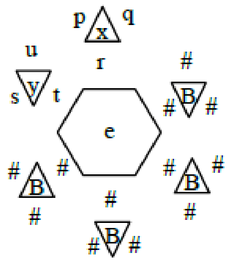
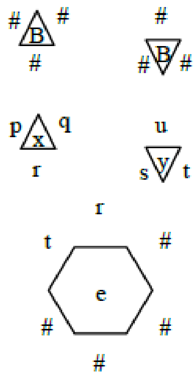
$\theta_5 =$  $: T_{\theta_5} =$ 

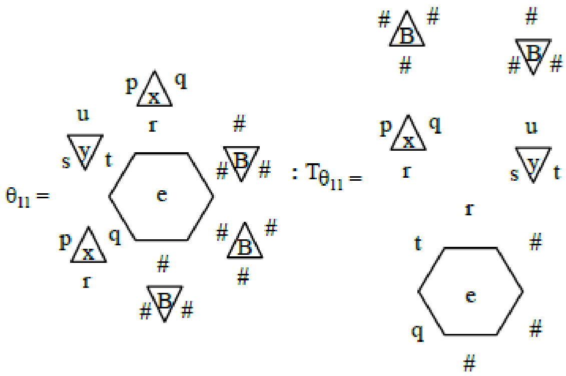
$\theta_6 =$  $: T_{\theta_6} =$ 

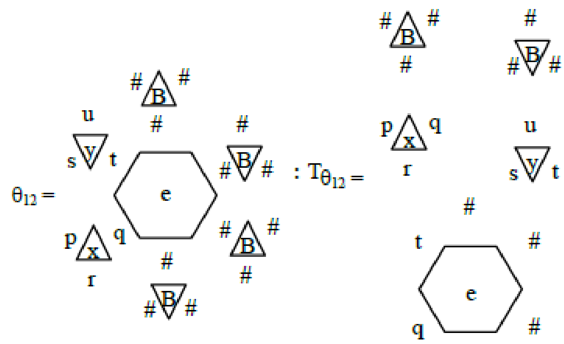
$\theta_7 =$  $: T_{\theta_7} =$ 

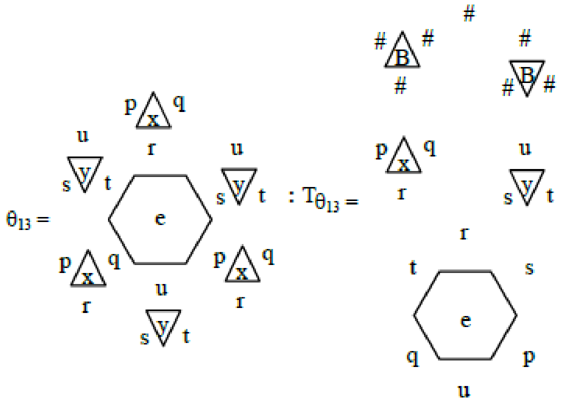
$\theta_8 =$  $: T_{\theta_8} =$ 

$\theta_9 =$  $: T_{\theta_9} =$ 

$\theta_{10} =$  $: T_{\theta_{10}} =$ 







Set $\Gamma = \{\triangle a, \nabla b\}$ $a, b \in \Sigma_1$ and $\theta = \{\theta_1, \theta_2, \theta_3 \ldots \theta_{13}\}$ is a finite set of Star of David tiles over the alphabet $\Gamma \cup \{\#\}$ and $\pi : \Gamma \to \Sigma_2$ is a projection. In fact we can take $L' = L(\theta)$ as an underlying local Oxide language, $L(\theta)$ is recognized by Oxide Wang system. Apply the projection $\pi : \Gamma \to \Sigma_2$. Then $L = \pi(L(\theta))$ is an Oxide tiling recognizable language over the alphabet Σ_2 .

CONCLUSION

In this paper we used two formalisms to recognize Oxide pictures. We started from local Oxide language, and then discussed recognizable Oxide picture language and Oxide tiling system. Moreover we introduced Oxide Wang system to recognize such Oxide pictures. Many language theoretic properties can be obtained relating to the classes.

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